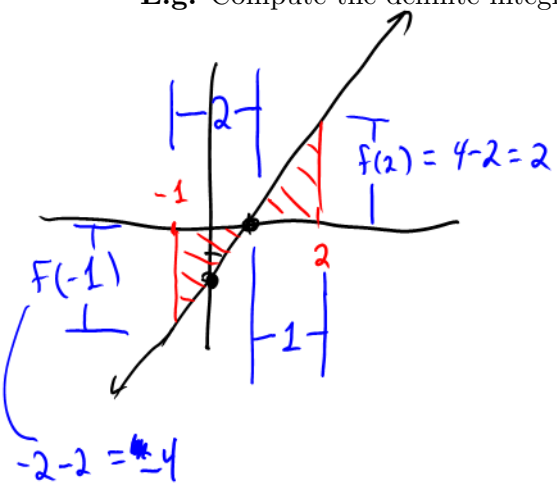
Computing Indefinite Integrals Geometrically

E.g. Compute the definite integral



$$\int_{-1}^{2}(2x-2)\,dx$$ = area above − area below

$$= \frac{1}{2}\cdot 1\cdot 2 - \frac{1}{2}\cdot 2\cdot 4$$

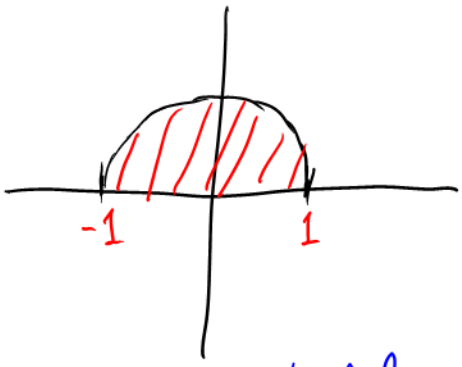
$$= 1 - 4$$

$$= -3$$

I can compute area of
① rectangles
② triangles
③ circles

E.g. Compute the definite integral

$$\int_{-1}^{1} \sqrt{1-x^2}\,dx$$



Remember $y^2 + x^2 = 1$ is a unit circle centered at $(0,0)$

So $y = \sqrt{1-x^2}$ is the top half of the circle

Notice: Shaded region is 1/2 of unit circle

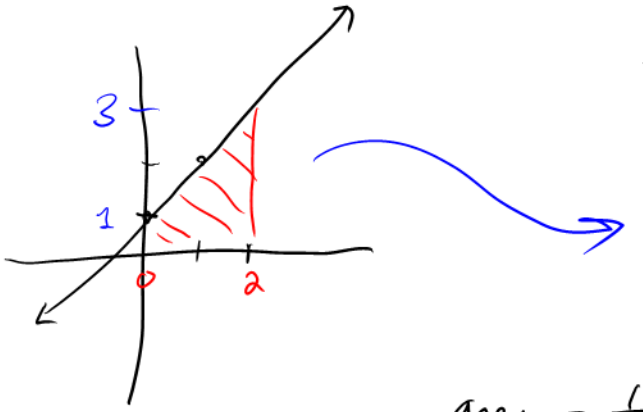
unit circle has area π

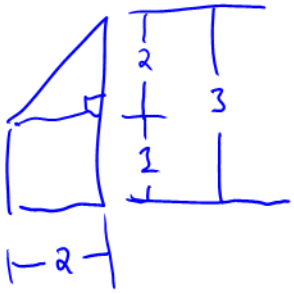
$\Rightarrow$ shaded region has area $\frac{1}{2}\pi$

$$\int_{-1}^{1} \sqrt{1-x^2}\,dx = \frac{1}{2}\pi$$

E.g. Compute the definite integral

$$\int_0^2 x + 1 \, dx$$





$$\text{area} = \frac{1}{2} \cdot 2 \cdot 2 + 2 \cdot 1$$

$$= 2 + 2$$

$$= 4$$

$$\int_0^2 (x+1)\, dx = 4$$